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APPLICATION OF LINEAR PROGRAMMING IN THE PLANNING OF ASSORTMENT

- USSR -

[Following is a translation of an article by L. Terekhov in the Russian-language journal Planovoye Khozyaystvo (Planned Economy) No.11, Moscow, 1962, pages 72-78.]

In recent years linear programming, as an effective method for resolving a wide range of planning-economic problems, has received not only recognition in principle, but also significant practical application. Although in practice only the most closely studied types of problems of linear programming have been coped with (mostly different variants of transport problems), the real economic effect from their resolution leaves no doubt about the great potential possibilities of this method. This is particularly clear if it is taken into consideration that the effect achieved as a result of linear programming does not, as a rule, require any kind of supplementary expenditures, but is a consequence of the better organization of production, fuller utilization of labor and material resources, and the perfection of methods and analysis.

The problem of the further elaboration and broad utilization

of linear programming requires not only the perfection of that method from the mathematical side, but also significant economic work directed primarily toward the formulation and analysis of new practical problems of optimum planning as well as the investigation of several general questions of an economic character such as the classification of tasks of linear programming, the selection and basing of criteria of optimality and others.

In works on linear programming, there is frequently formulated the so-called basic problem of productive planning with respect to which other problems are considered as particular cases. Such a formulation of the question is typical, for example, in the known works of L.V. Kantorovich /See L.V. Kantotovich, The Economic Calculation of the Best Utilization of Resources, USSR Academy of Sciences Press, 1960/. The basic problem of productive planning amounts to the following: there is a certain quantity of productive factors in limited volume; given different possible technological means of manufacturing products requiring unequal expenditures, the assortment of production turned out and its planning structure is finally known (the conditions of completeness). The intensity of application of different technological means is found by which, from the calculation of the limitability of factors, the maximum output of complete production will be secured. A different formulation of the problem is possible when the output of production is fixed not

only by structure, but also by absolute value; then the purpose of the computations is to obtain the given production with minimum expenditures. Using a somewhat broadened interpretation of the concept of "technological means", it is easily possible to bring under the stated general formulation such important particular problems as the best distribution of components among machine tools, where every pair of component-tools is considered as an individual technological means or transport problem, in which by technological means there is understood the transportation of freight from a definite supplier to a definite recipient and the quantity of freight indicates the intensity of the application of this or that means in the plan.

In the problems referred to, the given structure of production is the essential condition and the problem consists of such distribution of the tasks as will secure the best effect. But in the composition of the plan the structure of output far from always appears in the capacity of an evident or strictly determined characteristic. On the contrary, in a majority of cases it is exactly the structural composition of production which must in the first place be the object of optimization in order that the assortment and volume of output satisfy the requirements of the population and of the national economy to the greatest extent, and at the same time meet the potentialities of the given enterprise or production complex in the best way [Note: the circumstance that the nomenclature of production is established by

by the enterprise in the guise of directed tasks does not, of course, eliminate the necessity for the enterprises themselves as well as other planning organs to work at perfecting the planning of assortment in the process of drawing up the plan. This leads to another important group of problems of linear programming which can be designated as assortmental in contradistinction to distributional problems.

Different concrete forms of assortmental problems are known and there is a fairly significant experience in their resolution applicable to the activities of foreign firms. Under conditions of capitalism these problems have a very definite meaning: from a calculation of productive powers and potentialities for supplying finished products, it is required to establish the assortment and value of output securing the receipt of maximum profits. The problems of planning agricultural production have an analagous character, in which the optimum structure of areas sown to different crops is sought; since productivity in such problems is considered a known quantity, the determined structure of agricultural production simply corresponds to a certain structure of sown areas. Problems of formulating the optimum fodder ration, of mixing it and others adjoin this group.

Formally, the tasks of planning assortment as well as distributional tasks come under the general problem of productive planning. But with the presence of a certain community there exists a principal economic difference between these two tasks. In distributional

problems an alternative consists in the possibility of utilising different technological means as such, and if this or that means is shown to be relatively disadvantageous and does not get into the optimum plan, then there is no kind of obstacle to the rejection of that means. In assortmental problems alternative decisions do not have to do with technology but with production itself, and here it is completely clear that it is far from simple to decide the question of rejecting the production of less advantageous products or of the sharp decrease in its production. If we proceed from the economic content of the aforesaid tasks, then the difference between them is the difference between the problem of "what to produce" and the problem of "how to produce."

It is obvious that the planning of the structure of assortment presents itself as an independent problem the consideration of which, generally speaking, must be begun with the composition of the optimum plan. Moreover, in a sense the formulation and receipt of the initial figures in this problem is more simple than in distributional problems: it requires the knowledge of one technological process for each type of production (most frequently of the acting process), whereas in distributional problems precise figures on different possible technological means are necessary, frequently including those which are not used in the practice of production.

With comparative simplicity of its initial conditions, the

problem of planning of assortment moves into the not so easily resolved question of the criteria of optimality. A single and fully safe criterion for such problems is not difficult to formulate: the production of an enterprise must to the greatest extent serve to satisfy the general popular needs. However, in order to give this criterion numerical expression, it is necessary to co-measure /soizmerat'/ the consumption value of different types of production. The straight co-measurement of consumption values is generally impossible, since such an abstract measure of the usefulness of material wealth does not exist. In some cases products are co-measured according to certain most important consumption characteristics (for example, fodder crops according to the content of fodder units), but on the whole such a possibility is the exception rather than the rule.

On the other hand, not one of the criteria having a quantitative definition can be used without considerable reservations. Applications of criteria of maximum profits which are natural under conditions of capitalism are not, of course, excluded in a planned economy, but in this the decisive requirement is that the interests of securing the greatest profitability do not contradict the receipt of the production necessary to the national economy. In any case, it is impossible to speak of the unconditional safety of this method and the possibility of its utilization must be verified with the aid of methods of qualitative analysis.

The same roughly relates to another important valuative criterion, to the maximum of production in valued expression. With its use in the optimum plan there enter first of all those types of production which require relatively lower expenditures per unit of value. In the plan, as a rule, the better assimilated production gains an advantage over less assimilated production, that requiring the expenditure of more material gains an advantage over that requiring the expenditure of more labor, etc. It is obvious that such a plan will not always be acceptable for reasons of a qualitative nature.

The inadequacy of the stated criteria, which are capable of orienting the enterprise to the advantageous output of production which is profitable to it from the point of view of profits or "value" and to the detriment of assortment, has been long and well known. The perfection of exisiting value indices is necessary not only for the sake of broadening the bases for the utilization of mathematical methods, but also in the interest of resolving many other problems of economic analysis and planning. Nevertheless, even the most perfect value indices cannot, apparently, serve as the ideal criterion of optimality by relationship to qualitatively heterogeneous production.

All of this does not mean that a mathematical apparatus is not applicable in the planning of the volume and structure of production. Mathematical methods, and linear programming in particular, sharply broaden the possibilities of economic analysis and permit especial

experimentation on economo-mathematical models, fully sufficiently reflecting the connections and regularities of real activity. By utilizing different methods of formulating the task, using various criteria of optimality, correcting the initial conditions and limiting them in conformity with the factors of the previous stages of the experiment, it is possible to obtain a whole series of variants of the plan by methods of qualitative analysis to select one of them or even some intermediate variant which is practically the best not only according to the relationship to a particular criterion of optimality but also from the point of view of the main criterion, although this last does not have a strictly quantitative expression.

Even the most general formulation of the problem of planning the assortment permits various possibilities of searches for the optimum variant. We assume thatat an industrial enterprise, n different products can be produced. The enterprise disposes equipment, including m different groups, in each of which enter units of single-type equipment of approximately equal productivity (for example, groups of lathes, milling and grinding tools, etc.). For each type of production a definite technological process is known, characterized by the norms of time a_{ij} on processing a unit of goods i on equipment of the j group. Given the disposed fund of time of work of the equipment of each group (b_j). The unknown quantities of the problem (x_i) are the levels of production of each type of product.

The area of permissible solution of the problem is limited by the requirement that the general workload of the equipment by group not exceed the given fund of time of its work. In other words, for each of the m group of equipment, there must be fulfilled the conditions

$$\sum_{i=1}^{n} a_{ij} x_i \leq b_j \; (j = 1,2,3,\ldots,m).$$

In the most simple case of the stated conditions (together with the obvious condition of the non-negative significance of x_i) it is sufficient to resolve the problem at the optimum, or more precisely at the maximum, since in the given formulation the problem at the minimum loses its import. Profits or production in the valued calculation may serve as the criteria of maximization. The optimum variants accordingly show that limited volume of profit or production which can in general be achieved under the given conditions. There is no doubt that the presence of these characteristics has importance for the further work of composing the plan: at the very least, the possibility to judge how much the variant according to the level of profitability and volume of production definitively secured in the plan differs from the optimum in this relationship of variants is shown.

However, in the most simple scheme of the problem adduced, there are lacking whatever limitations may be laid on the demand

for production; in other words, it is assumed that for each type of production, the need significantly exceeds the capabilities of the enterprise. As a result, in the optimum variants there are shown as presented only some of the designations of products, and a majority of those remaining as "less advantageous" do not get into the plan. That solution, if it can have a practical significance, does so only in cases when the possibility of supply actually much exceeds the productive capacity of the enterprise and is the basis of specializing it in the production of a limited assortment of products.

Usually it is necessary to consider demand as the means of introducing supplementary limitations into the conditions of the problem. If the requirement and possibility of supply by individually designated production does not exceed the determined volume, then for the corresponding variables limitations are introduced from above; if, in addition, some other types of products must be turned out in quantities, then limitations are applied from below, not excluding, it would seem, the two-sided limitation of variables. Naturally, the introduction of supplementary limitations reduces freedom of selection in searching for optimum variants, but only with very rigid limitations is the area of permissible alternatives so narrow that it does not require special accounting methods for obtaining the best plan for satisfying all the initial conditions.

In any optimum plan reached by methods of linear programming, the most effective conditions of utilization of equipment and

other productive resources included in the problem are always secured. With such criteria of optimality as profits or value of production, the effectiveness of utilization of resources is understood, of course, from the point of view of exactly these criteria. However, the index of utilization of the means of production itself can serve as a criterion of optimality in the problem of productive planning. Thus, in a contemplated problem of planning assortment, a variant of the program of production can be obtained in which the general fund of time of work for all equipment is utilized to the greatest possible extent; in other words, underloading of equipment is minimal. This is achieved on account of the determination, by methods of linear programming, of such a composition of production turned out as corresponds in the best way to the composition and productive capacity of the equipment being applied.

Another possible formulation of the problem is related in that in a number of cases there exists not one variant with a maximal loading of equipment, but a multiplicity of variants in any of which full utilization of productive capacities is secured. When the requirement of full loading of equipment is introduced into the problem as a way of corresponding presentation of initial limitations, and for all or some groups of equipment instead of the inequalities

$$\sum_{i=1}^{n} a_{ij} x_i \leq b_j$$

there are applied equations of the type

$$\sum_{i=1}^{n} a_{ij} x_i = b_j.$$

it signifies that in the optimum plan equipment of corresponding groups must be fully utilized. Conditions of such a character are useful to apply, for example, in regard to the most productive, expensive or critical equipment.

Since in the given case the condition of maximal utilization of productive capability is not formally a criterion of optimality, but relates to the number of initial limitations, and since alternative variants of the solution exist, there is the possibility of making a calculation from the point of view of different criteria of effectiveness. Variants with maximum profit and the greatest level of production according to value can be obtained; in addition, with this new formulation, when at the extreme measure some of the limitations have a sort of equality, the problem can be resolved not only at the maximum but also at the minimum, for example, at the minimum expenditures of labor, critical materials, etc. In essence, to some extent two criteria are simultaneously applied to the problem, of which the second serves for the selection of one optimum variant among a series of variants which are optimum from the point of view of the first criterion, which is the full utilization of productive capabilities.

It is easy to see that even the most general type of problem of planning production permits such broad and various possibilities of analysis with the application of mathematical methods that there are created all the premises for forming an actually optimum plan, optimum exactly in the sense of the best combination of the general popular interests with the interests and capabilities of the enterprise.

As an example of the problem of forming an optimum assortment, there is given below an account composed from figures of one of the shops of a Moscow factory. The shop produces a locksmith instrument of the sixth designation which is basically earmarked for sale to the population.

In Table 1 there are shown the norms of time in minutes for processing units of each type of article on equipment of five groups (drilling, lathing, grinding, round milling and flat milling tools); there are also shown the figures on the disposed fund of time in hours for each group of tools.

TABLE 1.

Article	Group of Equipment				
	I	II	III	IV	V
A.....	7.5	1.8	3.8	1.5	10.1
B.....	0.7	0.6	0.4	-	2.3
C.....	16.0	7.2	3.3	6.4	0.6
D.....	2.0	0.8	0.9	0.4	-
E.....	4.1	0.2	2.0	0.3	1.9
F.....	0.6	0.2	2.9	-	2.0
Fund of time in Hours	40500	16200	20250	8100	16200

The problem of planning assortment, as is shown above, permits

different methods of formulation and various criteria of optimality. It represents the interest not only of the analysis of possible optimum variants but also their comparison with variants obtained without the application of linear programming. In Table 2 there are given four variants of the program for the problem considered. The first of these corresponds to the actual annual plan of the shop for 1963. The plan is somewhat lowered in comparison with the possibilities of the equipment, even if the structure of output envisioned in it does not deviate. For this reason another variant of the program is shown in the Table, which is calculated at maximum output with a reduction in the planned structure of assortment. Since alternative technological means are absent in the problem, the calculation of this variant is carried out with the aid of elementary examples.

The third and fourth variants of the program are obtained by the simplex method of linear programming. Profit is the criterion of optimality in both cases, but in calculating the third variant the initial limitations are given in the form of inequalities, and the fourth, in the form of equalities; that is, in the latter case there was posited the condition of full utilization of equipment of each of the five groups. Thus, mathematically, the conditions of the problem have the following aspect:

$$\sum a_{ij}x_i \leqslant b_j \quad \text{(and for the latter variant } \sum a_{ij}x_i = b_j\text{)}; \quad x_i \geqslant 0;$$

special function $\Sigma p_i x_i \rightarrow \max$

(p_i —— profit per unit of articles i).

For each of the programs in Table 2 along with the quantity of articles according to type there is given a summary index of: level of production in distribution prices of the enterprise, planned profit and utilization of equipment in percents of the general planned fund of time of its work.

It is easy to see that the planned structure of production allows only about a two-thirds loading of equipment, since with this structure one of the groups of equipment (in the given instance the third) is in a "narrow" place, and the possibilities of the remaining groups are to this or that extent underutilized. Linear programming gives that composition of production which to the greatest extent corresponds to the quantity and composition of the available equipment. A sharp (by 2.5 to 3 times) increase in the indices of production and profit in the two latter variants of the program in comparison with the two first is clarified not simply by the selection of the most profitable article, but by that selection of the structure of output which permits the full utilization of equipment and on account of this yields supplementary production. This is especially noticeable in the fourth variant, which contains almost all of the initial types of articles but on account of changes of structure in comparison with the plan ensures a significant improvement of all

summary indices.

TABLE 2.

VARIANTS OF THE ANNUAL PROGRAM OF PRODUCTION

Indices	By plan of enterprise	Maximum output in planned structure	Program for maximum profit	Program with full use of equipment
Quantity of articles (in thousand pieces)				
A.....	14,8	17,3	96,2	48,7
B.....	50,3	59,0	—	165,9
C.....	12,4	14,5	—	9,0
D.....	267,0	337,9	854,1	888,6
E.....	22,0	25,8	—	—
F.....	211,5	249,0	—	46,1
Production (in thousands of rubles)	2073,4	2433,6	5768,1	5739,2
Profits (in thousands of rubles)	461,1	541,4	1595,6	1381,0
Utilization of equipment (in %)	58,4	68,7	96,8	100,0

Analysis of the program with full utilization of equipment can unconditionally facilitate the discovery of latent reserves of increasing production, since frequently the irrational structure of products being produced causes a significant underloading of the capabilities of the enterprise. Even if in conditions of demand it is fully permissible to change assortment for improving the utilization of basic means and increasing the output of production,

the elementary methods of calculating do not permit this to be done, especially when the problem relates to the enterprise with different equipment and a large nomenclature of goods being produced. For this reason the setting of assortment and structure of production in conformity with the composition of equipment is one of the most valuable possibilities presented by linear programming.

It is not excluded, of course, that the optimum variants obtained in the example under consideration can be shown to be unacceptable, since the criterion of profit is not decisive in planning assortment. Practically, this is expressed in that some articles must be produced in greater or lesser quantity than is envisioned in the optimum variants obtained at the outset. In this case, as will be remembered, limitations can be laid on to the corresponding changes, and the optimum variants reconsidered with account of such limitations. Then the mathematical model is supplemented (to a severe degree for some goods) by limitations which may have the following type:

$$x_i \leq l_i$$
$$x_i \geq k_i$$
$$k_i \leq x_i \leq l_i$$

It is also useful to consider and analyze programs which are optimum in relation to other criteria, in particular the criteria of maximum goods production. Qualitative analysis of different variants in the finalaccount permits selection of the basic and acceptable resolution.

In estimating this or that variant of assortment tasks from the point of view of their corresponding demand and need in different types of production, it is necessary to a lesser degree to consider two obstacles. First, one and the same article may be produced, and as a rule, is factually produced not only in the given but in other enterprises. The relative "disadvantage" of some articles at a given definite enterprise does not at all signify that the same thing will take place in other production. It is sufficient to have another composition of equipment and another value by type of article -- and the optimum assortment is sharply altered. This gives the possibility by way of rational planning of assortment at a series of enterprises of one profile to obtain the improvement of productive indices without lowering the output of some articles, that is, one the whole to increase the production of all types of products taken into account.

Second, demand itself is not unchangeable, and its size does not, in the final analysis, depend on the level of prices. Optimization of assortment, as is seen from the given example, in and of itself leads to a definite increase in the profitability of production. In addition, premises are created for a definite lowering of the cost of production -- first of all on account of the better utilization of equipment. This causes the possibility for a lowering of prices especially of those goods the output of which is forced into conformity with the optimum plan. It is natural that in conditions of reduced prices the discover of supplementary needs

and an increase in demand can be expected. Influence in this or that direction on the structure of demand as a way of revising prices must of course be based in qualitative relationships.

The volume and assortment of products being produced relates to a number of the most important and definite indices in the plan of any enterprise. The definite interchangeability of individual products for their consumers, the presence of unsatisfactory demand on some means of production and articles of consumption and the possibility of the production of the same products by different enterprises cause a multiplicity of achievable variants in the determination of tasks of assortment. Investigation of these variants by methods of linear programming can render considerable help in the resolution of the following questions: specialization of enterprises in the production of products more closely corresponding to their resources; increases of profitability of production by way of determined changes of assortment (particularly applicable to planned-unprofitable enterprises); improvement of the utilization of basic means right up to the receipt of a program of production ensuring full loading of all equipment on hand.

On the whole the establishment of rational structures of production with the aid of mathematical methods will facilitate the improvement of the basic work indices of enterprises, including indices of cost and productivity of labor and the more complete satisfaction of demand in conformity with production.

-END-